

**SUGGESTED NEW STEPS TOWARD THE PRACTICAL  
IMPLEMENTATION OF COST-BENEFIT ANALYSIS**

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This paper is divided into two main parts. In Part I, I first present the case for using a shadow price of government funds as a standard component of cost-benefit analysis. I then go on to illustrate its actual use, dealing with a few standard public finance/government policy problems. In Part II I present a convenient short-cut method of extending a country's relevant real rate of return to reproducible capital, for use in deriving the discount rate (economic opportunity cost of capital) to be employed in cost-benefit analysis.

**I. Implementing A "Shadow Price of Government Funds"**

It is now a matter of decades since the concept of a shadow price of government funds started worming its way into our literature. It sort of got in by the back door, as a few brave souls started to estimate the "marginal cost of public funds". What they estimated was the cost of raising additional government revenue by adding to the rates of particular sets of taxes. This approach obviously led to different estimates of the marginal cost of public funds, depending on which tax rates one was thinking of increasing (not to mention the obvious fact that different

economists might use different models and/or make different assumptions about key parameters even in cases where they were exploring the effects of increasing just one particular tax.)

One definite effect of this spate of measurements of the marginal cost of public funds was its natural extension -- it only takes a small intellectual step to morph a “marginal cost of public funds” into a shadow price of the same. I think of the difference as follows: one can easily live quite happily with an extra dollar of revenue having a marginal cost of \$1.20 via raising cigarette taxes, of \$1.30 via raising the tax on telephone calls, of \$1.40 by raising the corporate income tax rate, and of \$1.50 by raising all personal income tax rates by a given percentage. We public finance economists know better than to think that our policymakers would be troubled by the evident fact that such differences reflect a degree of non-optimality of policy -- non-optimality is simply a fact characterizing all real-world tax structures and all changes in them.

The transition from a marginal cost to a shadow price of public funds would be easy if there were only one marginal cost to work from. But that is clearly not the case. In my own thinking about these issues my longtime answer was to throw up my hands in the face of the conceptually huge multiplicity of “marginal costs of public funds”, and to urge people to use the convention that the funds for the marginal outlay, the marginal project, the marginal program would always be “sourced” in the capital market. There, I repeatedly said, the same basic mechanism would be at work all the time -- additional pressure in the capital market would displace consumption and investment in proportions that were based in the relative elasticities of saving and investment to interest rates (and possible to other channels through which capital market pressure might work). I argued again and again that this idea of the government sourcing its funds in the capital market was not only convenient for us as cost-benefit analysts, but also had a degree of genuine realism, since day by day governments get their needed marginal cash

from the capital market, and typically allow any periodic cash surpluses to be reflected in the reduction of outstanding debt.

I lived quite happily with this idea of a conventional or canonical assumption of capital market sourcing until a certain moment in the late 1980s or the very early 1990s when a telephone call from Washington started me down a trail that ultimately shook me out of my complacency. The occasion was an upcoming mini-conference on cost-benefit methodology at the World Bank, to be attended by a number of experienced practitioners. My task was to write a piece about new challenges for the coming decade, which in principle seemed easy enough, but tucked into my “terms of reference” was a special request. I was supposed to try to help settle an argument that had arisen among World Bank economists dealing with cost-benefit analysis. One group, mindful of the growing literature on the marginal cost of public funds, was arguing strenuously for the implementation of a shadow price of government funds, while another group, mindful of the simplicity and usefulness of the convention of “sourcing” of funds in the capital market, was strenuously resisting the shadow-price idea. I seem to remember that the people who talked to me on this matter were from this latter group, and that they somehow expected that in the end I would come down on their side, rejecting the idea of a shadow price of government funds. And my vague recollection of these events is that at first I too expected that I would end up with the conclusion they expected. However, as things turned out, I ended up fully convinced that our methodology really did need a “shadow-price of government funds” -- however, it would not come instead of the conventional assumption of capital market sourcing, but rather as a necessary supplement to that assumption.

Let me now take you down the road that led to this unexpected conclusion. The first step was to imagine two projects, side-by-side as it were -- one dealing with an expansion of capacity

in a state-owned electricity project, the other dealing with a highway improvement on a road where no toll was (or was to be) collected. The profiles of benefits and costs of these two projects were to be identical, as we have conventionally measured them in the past (i.e., without applying any shadow price of government funds). The key difference between the projects was that in the case of the electricity projects, all the benefits were in the form of cash to the government, while in the case of the highway project, none of the benefits took this form. They consisted instead of the saving of time by the occupants of vehicles on the road, of savings of gasoline, tires, and vehicle wear-and-tear; these savings accruing not to the government but to the private owners of the vehicles.

This example provides the setting for the decisive intellectual experiment. For convenience, I assumed that the profile of benefits and costs, as described, fell just on the borderline of acceptability, with net present value equal to zero at the appropriate rate of discount (= economic opportunity cost of capital). And, of course, following convention, the projects were assumed to be financed in the capital market, via increases in the outstanding amount of government debt. Now we come to the big difference. In the case of the electricity project, with benefits all in cash, the sequence of flows of fiscal benefits over the project's economic life would end up paying in full, down to the last real penny, the debt incurred in the course of the project's construction. Not so for the highway project. Here the debt accumulated during construction would apparently never be paid. Instead, the government would have to go to the capital market each year to pay the interest on the debt, thus adding each year to its debt. By the time the project finally reached the end of its economic life, the government would face a huge project-induced debt. Under the previous conventional analysis (i.e., without a shadow price of government funds) there would be a huge accumulated debt at the end of highway project's life,

which would go on accumulating, forever.

I think it is pretty easy to see that this outcome is not acceptable. Somehow a way must be found to “close the books” on projects, all or part of whose benefits take forms other than cash flows into the government’s coffers. At this point, I decided (tentatively) to work with a new convention -- close the books at project’s end. In the case of the highway project, this would mean raising tax money in period N (end of project’s life) sufficient to pay off the full amount of debt accumulated on account of the project, over its entire lifetime.<sup>1</sup> And since this extra tax money would carry with it an extra cost (due to the incremental excess burden of taxation), this extra cost should be reflected in a shadow price of government funds that is greater than one dollar, for every incremental dollar that has to be raised.

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<sup>1</sup>This accumulation should not be done at the interest rate that the government pays on its debt, for the rate does not include the indirect cash-flow losses that are involved when the government goes into debt. Consider the following simple case. When anybody goes into the capital market to raise \$1000 in funds, those funds come at the expense of \$750 of displaced investment and of \$250 of displaced consumption (newly-stimulated saving). The gross-of-tax rate of return on investment is 12%, the corporate and property taxes thereon take half of this amount thus leaving a market rate of return of  $6 [= 12(1-.5)]$  percent. This market return is subject to a 33% personal income tax so that savers get a net-of-tax rate of return of  $4 [= 6(1-.33)]$  percent on their savings. Thus, when the government raises \$1,000 in capital market and pays a 6% rate on its bonds, that does not represent the full cash cost to the government. As a consequence of the extra \$1000 of debt the government has to pay \$60 per year in direct interest. It also loses \$45 per year in corporation taxes (on the \$750 of investment that was displaced, which would have yielded [at 12%] \$90 per year of taxable returns and \$45 per year of taxes. At the same time the government gains \$5 per year on the extra saving of \$250 that is stimulated by its borrowing. This comes from a return of \$15 received (at 6%) by the savers on this extra saving) which yields a flow of \$5 per year extra personal tax. So the net cash flow position of the government is an extra outlay of \$100 per year [= \$60 in interest plus \$45 in taxes foregone in lost investment minus \$5 in extra taxes received on newly stimulated savings]. This \$100 represents a cost of 10% (rather than the interest rate of 6%) on the extra \$1000 of government borrowing. It also represents in this case the standard opportunity cost of capital, which would typically be used to discount all flows of benefits and costs in any standard exercise in cost-benefit analysis.

The use of period N as the point in time when final accounts should be settled, as it were, was only an artifice -- a crutch to get us to see the underlying logic and motivation leading to the use of tax-raised funds to supplement those raised in the capital market. The bottom line was simply that somehow, the capital-market debt associated by each project should ultimately be paid, and that in cases where the cash flows to be generated by the project were not sufficient to accomplish this aim, tax-raised moneys should be called upon to do the job.

But tax-raised funds carry an excess burden, so when a project has to resort to taxes it logically should be charged with this excess burden. For the moment let us assume that the expected excess burden linked to raising an extra dollar via taxes is  $\lambda$ . Let  $C_{\$t}$  and  $B_{\$t}$  be the outlays and inflows of government cash each period. If the economic opportunity cost of capital is  $\omega$ , then at year N the amount to be raised by taxes would be

$\sum_{t=0}^N (C_{\$t} - B_{\$t})(1 + \omega)^{N-t}$ ,

and the extra charge for excess burden would be  $\lambda$  times this amount. Note, however, that

$$\lambda \sum_{t=0}^N (C_{\$t} - B_{\$t})(1 + \omega)^{N-t} \equiv \sum_{t=0}^N (\lambda C_{\$t} - \lambda B_{\$t})(1 + \omega)^{N-t}.$$

That is to say, it is not necessary for us to think in terms of settling accounts in period N, or in any other specific period. The cleanest, most straightforward way to take tax financing and the excess burden associated with it into account is to apply an extra charge or benefit of  $\lambda$  to each and every cash outflow or cash inflow from and to the public treasury, over the life of the project. Equivalently, it means multiplying every  $C_{\$t}$  and  $B_{\$t}$  by  $(1 + \lambda)$  as we build the time profile of cost and benefits over the entire life of each project. This is how  $(1 + \lambda)$  becomes the shadow price of government funds. It is a factor to be applied to each and every cash outlay or cash inflow of government money over each project's life, on a good or service, the increment to

efficiency cost is  $-T(\partial Q/\partial T)dT$  and the incremental revenue is

$$QdT + T(\partial Q/\partial T)dT.$$

If the supply of the good is infinitely elastic  $(\partial Q/\partial T) = (\partial Q/\partial p^d)$  along the demand curve for the good and the increment in efficiency cost per dollar of extra revenue is

$$\lambda = \frac{-T(\partial Q/\partial p^d)}{Q + T(\partial Q/\partial p^d)} = \frac{-T\eta}{p + T\eta} - \frac{-\tau\eta}{1 + \tau\eta},$$

where  $\eta(<0)$  is the price elasticity of demand for the good and  $\tau(=T/P)$  is the percentage rate of tax.

Thus for a broad-based tax like a value-added tax at the rate of 20%, we might have  $\eta = -.25$  and the excess burden per dollar of extra revenue would be  $.05/.95$ , or about 5%. But a 20% tax on a good with unitary demand elasticity would have  $\lambda = .20/.80 = .25$ . And an import tariff of 40% on a good whose import demand elasticity was  $-2$  would have  $\lambda = .80/.20 = 4.00$ . Note that the standard measure only makes sense when  $(\tau\eta) < 1$ ; where this inequality goes the other way we are on the far side of the Laffer (Dupuit) curve and one gets greater revenue by reducing rather than raising the tax rate.

In the case of a consumption tax the relevant distortion is that affecting labor supply. There the increase in efficiency cost stemming from an increase in the tax would be  $-T(\partial L/\partial T)dT$ , and the corresponding increment in tax revenue would be  $[L+T(\partial L/\partial T)]dT$ . Taking the ratio of these two expressions and using  $(\partial L/\partial T) = -(\partial L/\partial w)$ , we get the measure of  $\lambda$ , the increment of efficiency cost per extra dollar of revenue:

$$\lambda = \varepsilon\tau/(1-\varepsilon\tau),$$

where  $\varepsilon$  is the compensated elasticity of supply of labor.

This assumes that the labor in question faces a given wage in the market place -- i.e., an infinite elasticity of demand. The more general formula for efficiency cost per dollar of extra revenue is

$$\lambda = \frac{\tau\phi}{1 + \tau\phi},$$

where  $\phi (< 0)$  is equal to  $\varepsilon\eta/(\varepsilon-\eta)$  with  $\varepsilon > 0$  being the elasticity of supply, and  $\eta < 0$  being the elasticity of demand of the taxed item.

$\phi$  can be thought of as the percentage reduction in the quantity of a taxed item as the result of an increase in the tax wedge, where the amount of that increase is equal to 1% of the price of the taxed item. It is thus a sort of tax-elasticity, but it measures the response of quantity to a one-percentage-point rise in the tax rate (e.g., from 10 to 11 percent), not the response to a rise in that rate by 1% of itself (e.g., from 10 to 10.1 percent).

Our next step is to try to see what are the likely effects of actually putting to use the concept of a shadow price of government funds. We will do so with a series of standard public finance problems, so that readers can relate the new results to a familiar reference point in each case.

a) Efficiency Costs of a Tax ( $T_j$ ) Compared With Those of a Subsidy ( $Z_j$ ), where  $T_j \equiv Z_j$ . The standard result (exact for linear supply and demand functions), is that the efficiency cost of the tax is a triangle generated by inserting a tax wedge equal to  $T_j$ , to the left of the undistorted equilibrium point, while the efficiency cost of the subsidy is a similar triangle generated by inserting a subsidy wedge equal to  $Z_j$ , to the right of the undistorted equilibrium. Obviously, with linear supply and demand, and  $T_j = Z_j$ , the two efficiency cost triangles are equal.

But with a shadow price of public funds equal to  $(1+\lambda)$  the story is quite different. In the case of the tax we add a benefit equal to  $\lambda T_j X'$  thus the base of the efficiency cost triangle, while in the case of the subsidy we add a cost equal to  $\lambda Z_j X'$ , where  $X'$  is the new equilibrium quantity in the presence of the distortion.

b) The Optimal Level of a Tax  $T_j$ . In standard general-equilibrium theory, we learn that the optimal level  $\hat{T}_j$  of a tax on good  $j$  is defined by

$$\hat{T}_j (\partial X_j / \partial T_j) + \sum_{i \neq j} T_i^* (\partial X_i / \partial T_j) = 0.$$

Here  $\hat{T}_j$  is the optimal level of a new tax on good  $j$ , given that the pre-existing taxes  $T_i^*$  on other items do not change.

Note that in the above equation the first term picks up the incremental loss in revenue as  $T_j$  is marginally increased. What it says is that this incremental revenue loss in the market for  $X_j$  will at the optimal point be exactly offset by the induced incremental revenue gain  $\sum T_i^* \Delta X_j$  in the markets for other, already taxed items. The standard result ignores the incremental gain  $X_j \Delta T_j$ , because this represents a simple transfer -- as demand price goes up, demanders lose and government gains; as supply price goes down, suppliers lose and government gains. The winners and losers in these transfers are given equal weight (per dollar of gain or loss), so the gains and losses implicit in the term  $X_j \Delta T_j$  simply cancel each other.

This canceling no longer occurs when we have a shadow price of government funds  $> 0$ . Now there is a net gain of  $\lambda X_j \Delta T_j$ , and the new condition for the optimum tax  $\hat{T}_j$  becomes

$$\lambda X_j + \hat{T}_j (\partial X_j / \partial T_j)(1+\lambda) + \sum_{i \neq j} T_i^* (\partial X_i / \partial T_j)(1+\lambda) = 0$$

The optimal tax is obviously higher, once  $\lambda$  enters the picture (recall that  $\partial X_j / \partial T_j < 0$ ).

c) Optimal  $T_j$  as the Only Tax. Perhaps some insight can be gained from considering a simple case of a tax on a single commodity, with constant costs  $C_j$ . We know that a government that only cared about revenue (or a private monopolist) would set the tax so as to maximize

revenue at  $\frac{T_j}{C_j} = -1/\eta_j$  where  $\eta_j$  is the price elasticity of demand. At the same time a

government interested in pure efficiency would set  $T_j = 0$ .

Now the government in our case would consider efficiency and revenue so on  $T_j \Delta X_j$  (a loss on both counts) it would set a “price” of  $(1+\lambda)$ , while to  $X_j \Delta T_j$  it would ascribe a net benefit of  $\lambda$ , the net gains stemming from a “transfer” of this amount between demanders and government. The result of the optimizing calculation would be

$$\lambda X_j + (1+\lambda) \hat{T}_j (\partial X_j / \partial T_j) = 0.$$

This leads to  $\frac{\hat{T}_j}{C_j} = -\frac{\lambda}{\eta_j(1+\lambda)}$ . So our government would end up partway between the

“competitive” solution  $\hat{T}_j = 0$  and the “monopolistic” solution  $(\hat{T}_j / C_j) = -1/\eta_j$ .

d) The Case of an Optimal Subsidy. The standard case for a subsidy to  $X_j$  is the existence of a positive externality  $E_j$ . The optimal level of that subsidy in standard cost-benefit analysis would be  $\hat{Z}_j = E_j$ . What happens when we introduce a shadow price of government funds? We end up with a cost of

$$(1+\lambda) \hat{Z}_j (\partial X_j / \partial Z_j) dZ_j + \lambda X_j dZ_j,$$

and, of course, a benefit of  $E_j (\partial X_j / \partial Z_j) dZ_j$ . Setting benefit equal to cost, we get

$$(\hat{Z}_j / P_j) = (E_j / P_j)(1 + \lambda) - \lambda / \pi_j(1 + \lambda).$$

Here  $\pi_j = (\partial X_j / \partial Z_j)(P_j / X_j) > 0$ . Note that  $\pi(>0)$  is directly analogous (but for a subsidy) to  $\phi(<0)$ , which was defined earlier and applies to a tax. Obviously, the size of the optimal subsidy can be drastically cut by the simple introduction, even of a relatively modest (e.g.,  $1 + \lambda = 1.2$ ) shadow price of government funds.

e) Import Tariff versus a Production Subsidy for Import Substitute. Here the standard analysis is unequivocal for the normal, small-country (no monopoly or monopsony power) case. The efficiency cost of a tariff is the sum of a production cost and a consumption cost. One can get the production cost alone by introducing a subsidy to the domestic production of the tariffed item. One can get the consumption cost alone by imposing a tax on the consumption of the item, regardless of whether it is imported or domestically produced. Imposing a tariff at the rate  $T_j$  gives us the sum of these two costs (with the consumption tax and the production subsidy rates both equal to  $T_j$ ). Hence in the standard analysis a subsidy to domestic production is always better than (or at worst equal to) an import tariff at the same rate.

This is no longer the case when we introduce a shadow price of government funds. Now the cost of the subsidy (at the rate  $T_j$ ) is  $1/2T_j\Delta Q_j + \lambda Q_j T_j$ , where  $Q_j$  is domestic production, while the cost of the tariff is  $1/2T_j(\Delta Q_j - \Delta D_j) - \lambda(D_j - Q_j)T_j$ . Here  $D_j$  is total demand for good  $j$ , regardless of source -- obviously  $(\Delta Q_j - \Delta D_j)$  is equal to the change in the quantity of imports of  $j$ , occasioned by the tariff. Of course,  $\Delta Q_j > 0$  and  $\Delta D_j < 0$ .

So, yes, the pure direct efficiency cost of the tariff is still greater than that of a subsidy to domestic production alone, but the two adjustments stemming from the shadow price of government funds both work in favor of the tariff -- the tariff brings in cash, while the production

subsidy entails a cash outlay.

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I hope these few examples give readers a sense of how the introduction of a shadow price of government funds modifies our traditional analysis, and also of the mechanics involved in such an exercise.

## II. Some New “Tricks” For Estimating the Real Rate of Return to Capital

For a long time some of us have been using a particular “trick” to establish an initial value  $K_0$  for the reproducible capital stock of a country. This trick was the assumption that in the period “around”  $t = 0$ , the country’s reproducible capital stock was growing at the same rate as its real GDP -- i.e.,  $g_k = g_y$ ; or  $\Delta k/k_0 = \Delta y/y_0$ . We already have  $\Delta y/y_0$  from the national accounts, and we get  $\Delta K$  from  $I_g - \delta K_0$ , where  $I_g$  is gross investment and  $\delta$  is the assumed average rate of depreciation of reproducible capital. My own use of this trick goes back at least 30 years, and it is certainly quite possible that others were employing it even earlier.

Given how long the trick has been around, I was surprised recently to find that by making a certain rather natural extension of it, one could develop a rather quick and easy way to estimating the real rate of return itself. The basic equation that was used for the old trick was

$$(1) \quad I_{gt} = (\delta + g_k)K_{t-1}.$$

This simply defines  $g_k$ , the rate of growth of the capital stock. The trick, then, is the assumption that  $g_k = g_y$  for the period in question.

The new trick is simply to do the same sort of thing with the equation defining the share of capital in output

$$(2) \quad ay_t = (\rho + \delta)K_{t-1}.$$

Having  $\delta$  and  $K_{t-1}$  from (1) all we need is  $\alpha$ , an estimate of the share of reproducible capital in GDP in order to get an estimate of  $\rho$ , the net-of-depreciation rate of return to that subset of capital.

Let me take you through a hypothetical calculation. We take a period of time in a country in which the economy was moving along in a pretty normal way -- no big inflation problem, no major recession, no banking crisis, no asset price bubble or crash. All this in order to be able plausibly to assume that  $g_y = g_k$  over the period.

Then we turn to the estimation of  $\alpha$  the share of reproducible capital in GDP. For this we first deduct the national accounts data on wages and salaries from GDP. This is not enough, however, for the income accounts category “income of unincorporated enterprises”, including, of course, the self-employed, contains both income from labor and income from capital. We must find some way of estimating what part of this income “truly” belongs to labor, in order to exclude it from the calculation. Where the category of “income from unincorporated enterprises” is already a small fraction of GDP, a rough assumption -- such as labor’s share being 40% or 50% or 60% -- will probably be adequate. Where this category’s share is bigger, more work is necessary. Sometimes one can find census data identifying self-employed and family labor and delineating certain of their attributes. If, for example, their age and education levels are given, income could be imputed to them, according to the earnings of employed workers of similar age and education.

Next, we have the issue of income from land. This is somewhat complicated, because we must recognize that buildings, fences, roads, land-leveling, etc., are counted as reproducible capital (i.e., are included in gross investment in the national income accounts). So there is no real “natural” base at which we can actually observe the rents received by “pure land”. Another

problem in connection with land is that its economic return comes partly (in a growing economy) in the form of predictable capital gains, as land values rise with economic growth. These capital gains are not counted as part of a nation's GDP, hence we must recognize that what we are to exclude as returns to "pure land" is a figure substantially less than the owners' actual returns to owning the land. Trying to weave my way through this thicket of complications, I have come up with a formula that meets the test of plausibility, and that in numerous applications to date has not led to problems. That formula assigns to "pure land" one third of the value added in agriculture plus one tenth of the value added in the rental income from housing (which includes actual rents paid plus imputed rents from owner-occupied dwellings).

Let us assume that reproducible capital gets 40 to 50 percent of the GDP, once the above deductions (for family labor, self-employment, and "pure land") have been made that the rate of gross investment is 20% of GDP, that the relevant overall average depreciation rate is 4%, and that the rate of GDP growth over the chosen period is 3 to 4 percent.

For example, if the rate of growth of  $y$  is 3% and  $k$  is growing at the same rate, and  $I_g/y$  is 20%, then  $.20y = (.03+.04)K_{t-1}$ , and  $K_{t-1} = (.20/.07)y$ . Now if reproducible capital gets .40y (gross of depreciation), we have  $.40y = (\rho+.04)K_{t-1}$ . Combining these results we get

$$(\rho+.04) = .40y/((.20/.07)y) = .028/.20$$

$$\rho = .14 - .04 = 10\%$$

Taking another example, this time with the share of reproducible capital equal to 50% and the GDP growth rate equal to 4%, we have

$$.20y = (.04+.04)K_{t-1}, \text{ and}$$

$$.50y = (\rho+.04)K_{t-1}$$

$$(\rho+.04) = .50y/((.20/.08)y) = .20$$

$$\rho = .20 - .04 = 16\%.$$

The general formula for  $\rho$  is

$$(3) \quad \rho = [a(g_k + \delta)/s] - \delta, \quad \text{where } s = I_g/y.$$

Up to now we have assumed  $g_k = g_y$ , but note that equation (3) only contains  $g_k$ .

Hence we are not at all constrained to deal with periods for which we think  $g_k = g_y$ . We can have capital growing one or two points faster, or one or two points slower than  $y$ , depending on the general configuration of the growth process in the period in question. There follow a few examples.

Latin American countries coming out of a crisis period have relatively rapid GDP growth (say 6%) but quite normal investment rates (say  $s = 20\%$ ). Here it is reasonable to assume that GDP is growing, say, 2 percentage points faster than the capital stock. So here we might have

$$\rho = [.50(.04 + .04)/.20] - .04 = 16\%$$

In contrast, the Asian Tigers have had investment rates of over 30% of GDP during their periods of rapid growth, so here we can well expect that the capital stock is growing faster than GDP. If we take  $g_y = .08$ ,  $g_k = .10$ ,  $a = .50$ ,  $s = .35$ , we get

$$\rho = [.50(.04 + .10)/.35] - .04 = 16\%$$

Readers are invited to experiment with combinations of  $a$ ,  $s$ ,  $\delta$ , and  $g_k$  that they consider plausible. In my own experiments of this type I found it quite hard to end up with values for  $\rho$  outside the range of, say, 8% to 18%.

The U.S. case might be thought to be an exception, because of the relatively low share of reproducible capital in total GDP. But recall that the familiar figure of 25% for capital's share relates to net income from capital in the numerator and national income in the denominator.

Building a hypothetical U.S. reproducible capital stock based on 20% gross investment and a 3% growth rate, we would have  $.2y_t = (.04 + .03)K_{t-1}$  or  $K_{t-1} = 2.86y_t$ , depreciation equal to  $.1144y_t$ . If capital's net share in national income was 25%, its gross share in GDP would be  $36.44/111.44$  or about 33%. Taking  $a = .33$ ,  $g_k = .03$ ,  $s = .20$ , and  $\delta = .04$  we have, for a stylized U.S. case:

$$\rho = [.33(.04 + .03)/.20] - .04 = 7.55\%$$

With a little nudge to the depreciation rate to account for the higher than average fraction of vehicles and machinery in the reproducible capital stock, we would get

$$\rho = [.33[(.04 + .04)/.20] - .04 = 9.2\%$$

#### Accounting For Infrastructure Capital in Measuring the Rate of Return

For most countries the national accounts series on gross investment include both public and private investment, and public investment includes a considerable amount of capital that does not produce significant revenue -- public buildings, parks, nearly all roads, etc. The figures we have estimated above take the estimated cash return to revenue-producing reproducible capital (in the numerator) and express it as a fraction of the estimated value of all (revenue-producing plus non-revenue producing) capital (in the denominator). It is at least arguable that a more relevant figure for the marginal productivity of capital, for use in estimating a country's economic opportunity cost of capital would be obtained by taking the ratio of cash return to value of capital, both of these relating to just the revenue-producing part of the capital stock. This would entail leaving the infrastructure capital (i.e., the non-revenue-yielding part) out of the denominator as we calculate the rate of return.

It is not at all easy to estimate the amount of public investment that falls in the non-revenue-yielding category. For the present purpose I have used a study by Everhart and

Sumlinski.<sup>2</sup> as a guide. They too did not separate public investment into revenue-yielding and non-revenue-yielding categories, but their study covered 87 developing countries, and thus incorporated a lot of information. Public investment ranged to over 50% of total investment and up to over 25% of GDP. It is pretty clear that the countries in the high side of these ranges have quite a lot of public investment in the remunerative category -- electricity companies, factories, firms, etc. On the other hand countries at the low end of the range are presumed to have their public investment concentrated on the basics -- public buildings, roads, etc. Since these investments are likely to be present in all countries, I chose a figure that was in the lowest quartile of Everhart and Sumlinski's distributions, and assumed that public sector non-revenue-yielding investments represented a quarter of the total investment. For our "standard" case of total investment equal to 20% of GDP, this would give us non-revenue-yielding public investment of 5% of GDP. It turns out that this figure, too, is in the bottom quartile of Everhart and Sumlinski's distribution of the rate of public investment to GDP in their 87 countries.

Using the indicated assumption, one can only convert our earlier results into estimate of the rate of return to remunerative investments in a country. To do so, we just divide the earlier result by 0.75. Thus, where we had 10% as the return to total capital, we now get 13.33% as the return to remunerative investments, and where 16% was the earlier result the new result becomes 21.33%.

Which of these two concepts -- the rate of return to total capital or the rate of return to remuneration investments -- ought to enter into the calculation of the economic opportunity cost

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<sup>2</sup>Everhart, Stephen and Mariusz A. Sumlinski, 2001, Trends in Private Investment in Developing Countries: Statistics for 1970-2000 and the Impact on Private Investment of Corruption and the Quality of Public Investments, International Finance Corporation Discussion Paper No. 44, Washing, DC: The World Bank.

of capital to be used in cost-benefit analyses, is an open question. The key issue is the extent to which non-remuneration investment tend to be displaced as new demands for funds appear in the capital market.